東海大学紀要産業工学部２（2009年）25頁～34頁
Bull. School of Industrial Engineering Tokai Univ., 2 (2009) pp.25 -34

A method for determining intermediate eigensolutions of large, sparse, symmetric matrices by the double shifted inverse power method in the generalized eigenvalue problem

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(Received: 19 OCTOBER 2009, Accepted: 22 FEBRUARY 2010)

Abstract

The inverse power method is often used to solve large, sparse and symmetric eigenproblems. This paper presents a useful method on the double shifted inverse power method in the eigensolution of large sparse and symmetric matrices. The proposed method is a method requiring only a relatively small computational time which has the good accuracy and stability. The results obtained agree well with the exact solutions in short CPU time and this indicates that the proposed method provides efficient convergency.

1. INTRODUCTION

Numerical simulations in computational dynamics or other fields are often reduced to linear equations or eigenvalue problems with large, sparse and symmetric matrices. These are usually solved with preconditioned iterative conjugate gradient (PCG) methods such as incomplete Cholesky conjugate gradients (ICCG) and scaled conjugate gradients (SCG), combined with algebraic multigrid (AMG) preconditioners [1,2,4,7,12,13]. Because these require much less use of the main storage area in memory, they have proven to be more powerful analytical approaches for the large sparse symmetric matrices created using the finite element method (FEM). Still, eigenvalue analysis is an ill-conditioned numerical method employing determinant of a matrix with zero, and so solution methods dependent on diagonally dominant matrices such as AMG and ICCG are not very effective for identifying intermediate eigenvalues. To make use of the sparsity of the matrices appearing in large, sparse and symmetric eigenvalue problems such as FEM, one must employ iterative methods such as power method solvers or Lanczos solvers [2,3,5,6,9,10, 11,14,15]. However, none of these solutions, except the inverse power method, is effective for finding intermediate eigenvalues.

Iterative methods such as the inverse power method, the subspace methods and the restart Lanczos method [2] are effective for finding intermediate eigenvalues after the minimum or maximum eigenvalue is identified [9,10,11], but they present insoluble or overlooked intermediate eigensolutions in the vicinity of the shifted origin. Because there is no appropriate way to calculate intermediate eigenvalues, the current approach is to start with the minimum and calculate “up” some hundreds or thousands of eigenvalues. This kind of analysis imposes an immense calculation load, and is only considered efficient for calculating some tens (at most) of the eigensolutions in the neighborhood of any one known eigenvalue.

Moving the origin to the neighborhood of an eigenvalue to search for more eigenvalues is called the *shifted origin inverse power method*. Once the shift distance is set correctly, we can expect calculations to converge more quickly than they would if the origin had been left in its original position. It is a classic

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